

Bounds on the magnetic moment and electric dipole moment of the τ -neutrino via the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$

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Abstract

Bounds on the anomalous magnetic moment and the electric dipole moment of the τ -neutrino are calculated through the reaction $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ at the Z_1 -pole, and in the framework of a left-right symmetric model. The results are based on the recent data reported by the L3 Collaboration at CERN LEP. We find that the bounds are almost independent of the mixing angle ϕ of the model in the allowed experimental range for this parameter. In addition, the analytical and numerical results for the cross-section have never been reported in the literature before.

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I. INTRODUCTION

In many extensions of the Standard Model (SM) the neutrino acquires a nonzero mass, a magnetic moment and an electric dipole moment [1]. In this manner the neutrinos seem to be likely candidates for carrying features of physics beyond the Standard Model [2]. Apart from masses and mixings, magnetic moments and electric dipole moments are also signs of new physics and are of relevance in terrestrial experiments, in the solar neutrino problem, in astrophysics and in cosmology [3].

At the present time, all of the available experimental data for electroweak processes can be understood in the context of the Standard Model of the electroweak interactions (SM) [2], with the exception of the results of the SUPER-KAMIOKANDE experiment on the neutrino-oscillations [4], as well as the GALLEX, SAGE, GNO, HOMESTAKE and LSND [5] experiments. Nonetheless, the SM is still the starting point for all the extended gauge models. In other words, any gauge group with physical characteristics must have as a subgroup the $SU(2)_L \times U(1)$ group of the standard model in such a way that their predictions agree with those of the SM at low energies. The purpose of the extended theories is to explain some fundamental aspects which are not clarified in the frame of the SM. One of these aspects is the origin of the parity violation at current energies. The Left-Right Symmetric Models (LRSM), based on the $SU(2)_R \times SU(2)_L \times U(1)$ gauge group [6], give an answer to this problem by restoring the parity symmetry at high energies and giving their violations at low energies as a result of the breaking of gauge symmetry. Detailed discussions on LRSM can be found in the literature [7–9].

In 1994, T. M. Gould and I. Z. Rothstein [10] reported a bound on the tau neutrino magnetic moment which they obtained through the analysis of the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$, near the Z_1 -resonance, by considering a massive tau neutrino and using Standard Model $Z_1e^+e^-$ and $Z_1\nu\bar{\nu}$ couplings.

At low center of mass energy $s \ll M_{Z_1}^2$, the dominant contribution to the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ involves the exchange of a virtual photon [11]. The dependence on the magnetic moment

comes from a direct coupling to the virtual photon, and the observed photon is a result of initial state Bremsstrahlung.

At higher s , near the Z_1 pole $s \approx M_{Z_1}^2$, the dominant contribution for $E_\gamma > 10 \text{ GeV}$ [12] involves the exchange of a Z_1 boson. The dependence on the magnetic moment and the electric dipole moment now comes from the radiation of the photon observed by the neutrino or antineutrino in the final state. The Feynman diagrams which give the most important contribution to the cross section are shown in Fig. 1. We emphasize here the importance of the final state radiation near the Z_1 pole, which occurs preferentially at high E_γ compared to conventional Bremsstrahlung.

Our aim in this paper is to analyze the reaction $e^+e^- \rightarrow \nu\bar{\nu}\gamma$. We use recent data collected with the L3 detector at CERN LEP [12–15] near the Z_1 boson resonance in the framework of a left-right symmetric model and we attribute an anomalous magnetic moment and an electric dipole moment to a massive tau neutrino. Processes measured near the resonance serve to set limits on the tau neutrino magnetic moment and electric dipole moment. In this paper, we take advantage of this fact to set bounds for μ_{ν_τ} and d_{ν_τ} for different values of the mixing angle ϕ [16–18], which is consistent with other constraints previously reported [10,11,15,19,22] and [20,21].

We do our analysis near the resonance of the Z_1 ($s \approx M_{Z_1}^2$). Thus, our results are independent of the mass of the additional heavy Z_2 gauge boson which appears in these kinds of models. Therefore, we have the mixing angle ϕ between the left and the right bosons as the only additional parameter besides the SM parameters.

The L3 Collaboration evaluated the selection efficiency using detector-simulated $e^+e^- \rightarrow \nu\bar{\nu}\gamma(\gamma)$ events, random trigger events, and large-angle $e^+e^- \rightarrow e^+e^-$ events. A total of 14 events were found by the selection. The distributions of the photon energy and the cosine of its polar angle are consistent with SM predictions. The total number of events expected from the SM is 14.1. If the photon energy is greater than half the beam energy, 2 events are selected from the data and 2.4 events are expected from the SM in the $\nu\bar{\nu}\gamma$ channel.

This paper is organized as follows: In Sect. II we describe the model with the Higgs sector

having two doublets and one bidoublet. In Sect. III we present the calculus of the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$. In Sect. IV we make the numerical computations. Finally, we summarize our results in Sect. V.

II. THE LEFT-RIGHT SYMMETRIC MODEL (LRSM)

We consider a Left-Right Symmetric Model (LRSM) consisting of one bidoublet Φ and two doublets χ_L, χ_R . The vacuum expectation values of χ_L, χ_R break the gauge symmetry to give mass to the left and right heavy gauge bosons. This is the origin of the parity violation at low energies [7], *i.e.*, at energies produced in actual accelerators. The Lagrangian for the Higgs sector of the LRSM is [8]

$$\mathcal{L}_{LRSM} = (D_\mu\chi_L)^\dagger(D^\mu\chi_L) + (D_\mu\chi_R)^\dagger(D^\mu\chi_R) + Tr(D_\mu\Phi)^\dagger(D^\mu\Phi). \quad (1)$$

The covariant derivatives are written as

$$\begin{aligned} D_\mu\chi_L &= \partial_\mu\chi_L - \frac{1}{2}ig\tau \cdot \mathbf{W}_L\chi_L - \frac{1}{2}ig'B\chi_L, \\ D_\mu\chi_R &= \partial_\mu\chi_R - \frac{1}{2}ig\tau \cdot \mathbf{W}_R\chi_R - \frac{1}{2}ig'B\chi_R, \\ D_\mu\Phi &= \partial_\mu\Phi - \frac{1}{2}ig(\tau \cdot \mathbf{W}_L\Phi - \Phi\tau \cdot \mathbf{W}_R). \end{aligned} \quad (2)$$

In this model there are seven gauge bosons: the charged $W_{L,R}^1, W_{L,R}^2$ and the neutral $W_{L,R}^3, B$. The gauge couplings constants g_L and g_R of the $SU(2)_L$ and $SU(2)_R$ subgroups respectively, are equal: $g_L = g_R = g$, since manifest left-right symmetry is assumed [23]. g' is the gauge coupling for the $U(1)$ group.

The transformation properties of the Higgs bosons under the group $SU(2)_L \times SU(2)_R \times U(1)$ are $\chi_L \sim (1/2, 0, 1)$, $\chi_R \sim (0, 1/2, 1)$ and $\Phi \sim (1/2, 1/2^*, 0)$. After spontaneous symmetry breaking, the ground states are of the form

$$\langle\chi_L\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_L \end{pmatrix}, \quad \langle\chi_R\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_R \end{pmatrix}, \quad \langle\Phi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix}, \quad (3)$$

breaking the symmetry group to form $U(1)_{em}$ giving mass to the gauge bosons and fermions, with the photon remaining massless. In Eq. (3), v_L , v_R , k and k' are the vacuum expectation values. The part of the Lagrangian that contains the mass terms for the charged boson is

$$\mathcal{L}_{mass}^C = (W_L^+ \ W_R^+) M^C \begin{pmatrix} W_L^- \\ W_R^- \end{pmatrix}, \quad (4)$$

where $W^\pm = \frac{1}{\sqrt{2}}(W^1 \mp W^2)$.

The mass matrix M^C is

$$M^C = \frac{g^2}{4} \begin{pmatrix} v_L^2 + k^2 + k'^2 & -2kk' \\ -2kk' & v_R^2 + k^2 + k'^2 \end{pmatrix}. \quad (5)$$

This matrix is diagonalized by an orthogonal transformation which is parametrized [23] by an angle ζ . This angle has been restricted to have a very small value because of the hyperon β decay data [24].

Similarly, the part of the Lagrangian that contains the mass terms for the neutral bosons is

$$\mathcal{L}_{mass}^N = \frac{1}{8} (W_L^3 \ W_R^3 \ B) M^N \begin{pmatrix} W_L^3 \\ W_R^3 \\ B \end{pmatrix}, \quad (6)$$

where the matrix M^N is given by

$$M^N = \frac{1}{4} \begin{pmatrix} g^2(v_L^2 + k^2 + k'^2) & -g^2(k^2 + k'^2) & -gg'v_L^2 \\ -g^2(k^2 + k'^2) & g^2(v_R^2 + k^2 + k'^2) & -gg'v_R^2 \\ -gg'v_L^2 & -gg'v_R^2 & g'^2(v_L^2 + v_R^2) \end{pmatrix}. \quad (7)$$

Since the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ is neutral, we center our attention on the mass terms of the Lagrangian for the neutral sector, Eq. (6).

The matrix M^N for the neutral gauge bosons is diagonalized by an orthogonal transformation which can be written in terms of the angles θ_W and ϕ [25]

$$U^N = \begin{pmatrix} c_W c_\phi & -s_W t_W c_\phi - r_W s_\phi / c_W & t_W (s_\phi - r_W c_\phi) \\ c_W s_\phi & -s_W t_W s_\phi + r_W c_\phi / c_W & -t_W (c_\phi + r_W s_\phi) \\ s_W & s_W & r_W \end{pmatrix}, \quad (8)$$

where $c_W = \cos \theta_W$, $s_W = \sin \theta_W$, $t_W = \tan \theta_W$ and $r_W = \sqrt{\cos 2\theta_W}$, with θ_W being the electroweak mixing angle. Here, $c_\phi = \cos \phi$ and $s_\phi = \sin \phi$. The angle ϕ is considered as the angle that mixes the left and right handed neutral gauge bosons $W_{L,R}^3$. The expression that relates the left and right handed neutral gauge bosons $W_{L,R}^3$ and B with the physical bosons Z_1 , Z_2 and the photon is:

$$\begin{pmatrix} Z_1 \\ Z_2 \\ A \end{pmatrix} = U^N \begin{pmatrix} W_L^3 \\ W_R^3 \\ B \end{pmatrix}. \quad (9)$$

The diagonalization of (5) and (7) gives the mass of the charged $W_{1,2}^\pm$ and neutral $Z_{1,2}$ physical fields:

$$M_{W_{1,2}}^2 = \frac{g^2}{8} [v_L^2 + v_R^2 + 2(k^2 + k'^2) \mp \sqrt{(v_R^2 - v_L^2)^2 + 16(kk')^2}], \quad (10)$$

$$M_{Z_1, Z_2}^2 = B \mp \sqrt{B^2 - 4C}, \quad (11)$$

respectively, with

$$B = \frac{1}{8} [(g^2 + g'^2)(v_L^2 + v_R^2) + 2g^2(k^2 + k'^2)],$$

$$C = \frac{1}{64} g^2 (g^2 + 2g'^2) [v_L^2 v_R^2 + (k^2 + k'^2)(v_L^2 + v_R^2)].$$

Taking into account that $M_{W_2}^2 \gg M_{W_1}^2$, from the expressions for the masses of M_{Z_1} and M_{Z_2} , we conclude that the relation $M_{W_1}^2 = M_{Z_1}^2 \cos^2 \theta_W$ still holds in this model.

From the Lagrangian of the LRSM, we extract the terms for the neutral interaction of a fermion with the gauge bosons $W_{L,R}^3$ and B :

$$\mathcal{L}_{int}^N = g(J_L^3 W_L^3 + J_R^3 W_R^3) + \frac{g'}{2} J_Y B. \quad (12)$$

Specifically, the interaction Lagrangian for $Z_1 \rightarrow f\bar{f}$ [26] is

$$\mathcal{L}_{int}^N = \frac{g}{c_W} Z_1 [(c_\phi - \frac{s_W^2}{r_W} s_\phi) J_L - \frac{c_W^2}{r_W} s_\phi J_R], \quad (13)$$

where the left (right) current for the fermions are

$$J_{L,R} = J_{L,R}^3 - \sin^2 \theta_W J_{em},$$

and

$$J_{em} = J_L^3 + J_R^3 + \frac{1}{2} J_Y$$

is the electromagnetic current. From (13) we find that the amplitude \mathcal{M} for the decay of the Z_1 boson with polarization ϵ^λ into a fermion-antifermion pair is:

$$\mathcal{M} = \frac{g}{c_W} [\bar{u} \gamma^\mu \frac{1}{2} (a g_V - b g_A \gamma_5) v] \epsilon_\mu^\lambda, \quad (14)$$

with

$$a = c_\phi - \frac{s_\phi}{r_W} \quad \text{and} \quad b = c_\phi + r_W s_\phi, \quad (15)$$

where ϕ is the mixing parameter of the LRSM [16,17].

In the following section, we make the calculations for the reaction $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ by using the expression (14) for the transition amplitude.

III. THE TOTAL CROSS SECTION

We calculate the total cross section of the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ using the Breit-Wigner resonance form [27,28]

$$\sigma(e^+e^- \rightarrow \nu\bar{\nu}\gamma) = \frac{4\pi(2J+1)\Gamma_{e^+e^-}\Gamma_{\nu\bar{\nu}\gamma}}{(s - M_{Z_1}^2)^2 + M_{Z_1}^2\Gamma_{Z_1}^2}, \quad (16)$$

where $\Gamma_{e^+e^-}$ is the decay rate of Z_1 to the channel $Z_1 \rightarrow e^+e^-$ and $\Gamma_{\nu\bar{\nu}\gamma}$ is the decay rate of Z_1 to the channel $Z_1 \rightarrow \nu\bar{\nu}\gamma$.

In the next subsection we calculate the widths of Eq. (16).

A. Width of $Z_1 \rightarrow e^+e^-$

In this section we calculate the total width of the reaction

$$Z_1 \rightarrow e^+e^-, \quad (17)$$

in the context of the left-right symmetric model which is described in Section II.

The expression for the total width of the process $Z_1 \rightarrow e^+e^-$, due only to the Z_1 boson exchange, according to the diagrams depicted in Fig. 1, and using the expression for the amplitude given in Eq. (14), is:

$$\Gamma_{(Z_1 \rightarrow e^+e^-)} = \frac{G_F M_{Z_1}^3}{6\pi\sqrt{2}} \sqrt{1 - 4\eta} [a^2 (g_V^e)^2 (1 + 2\eta) + b^2 (g_A^e)^2 (1 - 4\eta)], \quad (18)$$

where $\eta = m_e^2/M_{Z_1}^2$.

We take $g_V^e = -\frac{1}{2} + 2\sin^2\theta_W$ and $g_A^e = -\frac{1}{2}$ from the experimental data [27], so that the total width with $m_e = 0$ is

$$\Gamma_{(Z_1 \rightarrow e^+e^-)} = \frac{\alpha M_{Z_1}}{24} \left[\frac{\frac{1}{2}(a^2 + b^2) - 4a^2 x_W + 8a^2 x_W^2}{x_W(1 - x_W)} \right], \quad (19)$$

where $x_W = \sin^2\theta_W$ and $\alpha = e^2/4\pi$ is the fine structure constant.

B. Width of $Z_1 \rightarrow \nu\bar{\nu}\gamma$

The expression for the Feynman amplitude \mathcal{M} of the process $Z_1 \rightarrow \nu\bar{\nu}\gamma$ is due only to the Z_1 boson exchange, as shown in the diagrams in Fig. 1. We use the expression for the amplitude given in Eq. (14) and assume that a massive Dirac neutrino is characterized by the following phenomenological parameters: a charge radius $\langle r^2 \rangle$, a magnetic moment $\mu_{\nu\tau} = \kappa\mu_B$ (expressed in units of the Bohr magneton μ_B) and an electric dipole moment $d_{\nu\tau}$. Therefore, the expression for the Feynman amplitude \mathcal{M} of the process $Z_1 \rightarrow \nu\bar{\nu}\gamma$ is given by

$$\mathcal{M}_a = [\bar{u}(p_\nu)\Gamma^\alpha \frac{i}{(\ell - m_\nu)} (-\frac{ig}{4\cos\theta_W} \gamma^\beta (a - b\gamma_5)) v(p_{\bar{\nu}})] \epsilon_\alpha^\lambda(\gamma) \epsilon_\beta^\lambda(Z_1) \quad (20)$$

and

$$\mathcal{M}_b = [\bar{u}(p_\nu)(-\frac{ig}{4\cos\theta_W}\gamma^\beta(a - b\gamma_5))\frac{i}{(\not{k} - m_\nu)}\Gamma^\alpha v(p_{\bar{\nu}})]\epsilon_\alpha^\lambda(\gamma)\epsilon_\beta^\lambda(Z_1), \quad (21)$$

where

$$\Gamma^\alpha = eF_1(q^2)\gamma^\alpha + \frac{ie}{2m_\nu}F_2(q^2)\sigma^{\alpha\mu}q_\mu + eF_3(q^2)\gamma_5\sigma^{\alpha\mu}q_\mu, \quad (22)$$

is the neutrino electromagnetic vertex, e is the charge of the electron, q^μ is the photon momentum and $F_{1,2,3}(q^2)$ are the electromagnetic form factors of the neutrino, corresponding to charge radius, magnetic moment and electric dipole moment, respectively, at $q^2 = 0$ [20,29]. While ϵ_α^λ and ϵ_β^λ are the polarization vectors of photon and of the boson Z_1 , respectively. $l(k)$ stands by the momentum of the virtual neutrino (antineutrino), and the coupling constants a and b are given in the Eq. (15).

After applying some of the theorems of traces of the Dirac matrices and of sum and average over the initial and final spins, the square of the matrix elements becomes

$$\sum_s |\mathcal{M}_T|^2 = \frac{g^2}{4\cos^2\theta_W}(\mu_{\nu_\tau}^2 + d_{\nu_\tau}^2)[(a^2 + b^2)(s - 2\sqrt{s}E_\gamma) + a^2E_\gamma^2\sin^2\theta_\gamma]. \quad (23)$$

Our following step, now that we know the square of the Eq. (23) transition amplitude, is to calculate the total width of $Z_1 \rightarrow \nu\bar{\nu}\gamma$:

$$\Gamma_{(Z_1 \rightarrow \nu\bar{\nu}\gamma)} = \int \frac{\alpha(\mu_{\nu_\tau}^2 + d_{\nu_\tau}^2)}{96\pi^2 M_{Z_1} x_W (1 - x_W)} [(a^2 + b^2)(s - 2\sqrt{s}E_\gamma) + a^2E_\gamma^2\sin^2\theta_\gamma] E_\gamma dE_\gamma d\cos\theta_\gamma, \quad (24)$$

where E_γ and $\cos\theta_\gamma$ are the energy and scattering angle of the photon.

The substitution of (19) and (24) in (16) gives

$$\begin{aligned} \sigma(e^+e^- \rightarrow \nu\bar{\nu}\gamma) = & \int \frac{\alpha^2(\mu_{\nu_\tau}^2 + d_{\nu_\tau}^2)}{192\pi} \left[\frac{\frac{1}{2}(a^2 + b^2) - 4a^2x_W + 8a^2x_W^2}{x_W^2(1 - x_W)^2} \right. \\ & \left. \left[\frac{(a^2 + b^2)(s - 2\sqrt{s}E_\gamma) + a^2E_\gamma^2\sin^2\theta_\gamma}{(s - M_{Z_1}^2)^2 + M_{Z_1}^2\Gamma_{Z_1}^2} \right] E_\gamma dE_\gamma d\cos\theta_\gamma. \right. \end{aligned} \quad (25)$$

It is useful to consider the smallness of the mixing angle ϕ , as indicated in the Eq. (29), to approximate the cross section in (25) by its expansion in ϕ up to the linear term:

$\sigma = (\mu_{\nu_\tau}^2 + d_{\nu_\tau}^2)[A + B\phi + O(\phi^2)]$, where A and B are constants which can be evaluated. Such an approximation for deriving the bounds of μ_{ν_τ} and d_{ν_τ} is more illustrative and easier to manipulate.

For $\phi < 1$, the total cross section for the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ is given by

$$\sigma(e^+e^- \rightarrow \nu\bar{\nu}\gamma) = (\mu_{\nu_\tau}^2 + d_{\nu_\tau}^2)[A + B\phi + O(\phi^2)], \quad (26)$$

where explicitly A is:

$$A = \int \frac{\alpha^2}{96\pi} \left[\frac{1 - 4x_W + 8x_W^2}{x_W^2(1 - x_W)^2} \right] \left[\frac{s - 2\sqrt{s}E_\gamma + \frac{1}{2}E_\gamma^2 \sin^2 \theta_\gamma}{(s - M_{Z_1}^2)^2 + M_{Z_1}^2 \Gamma_{Z_1}^2} \right] E_\gamma dE_\gamma d\cos \theta_\gamma, \quad (27)$$

while B is given by

$$B = \int \frac{\alpha^2}{48\pi} \left\{ \left[\frac{3 - 8x_W}{r_W x_W (1 - x_W)^2} \right] \left[\frac{s - 2\sqrt{s}E_\gamma + \frac{1}{2}E_\gamma^2 \sin^2 \theta_\gamma}{(s - M_{Z_1}^2)^2 + M_{Z_1}^2 \Gamma_{Z_1}^2} \right] \right. \\ \left. - \left[\frac{1 - 4x_W + 8x_W^2}{r_W x_W (1 - x_W)^2} \right] \left[\frac{x_W(s - 2\sqrt{s}E_\gamma) + \frac{1}{2}E_\gamma^2 \sin^2 \theta_\gamma}{(s - M_{Z_1}^2)^2 + M_{Z_1}^2 \Gamma_{Z_1}^2} \right] \right\} E_\gamma dE_\gamma d\cos \theta_\gamma. \quad (28)$$

The expression given for A corresponds to the cross section previously reported by T.M. Gould and I.Z. Rothstein [10], while B comes from the contribution of the LRSM.

Evaluating the limit when the mixing angle is $\phi = 0$, the second term in (26) is zero and Eq. (26) is reduced to the expression (3) given in Ref. [10].

IV. RESULTS

In order to evaluate the integral of the total cross section as a function of mixing angle ϕ , we require cuts on the photon angle and energy to avoid divergences when the integral is evaluated at the important intervals of each experiment. We integrate over θ_γ from 44.5° to 135.5° and E_γ from 15 GeV to 100 GeV for various fixed values of the mixing angle $\phi = -0.009, -0.005, 0, 0.004$. Using the following numerical values: $\sin^2 \theta_W = 0.2314$, $M_{Z_1} = 91.187 \text{ GeV}$, $\Gamma_{Z_1} = 2.49 \text{ GeV}$, we obtain the cross section $\sigma = \sigma(\phi, \mu_{\nu_\tau}, d_{\nu_\tau})$.

For the mixing angle ϕ between Z_1 and Z_2 , we use the reported data of M. Maya *et al.* [16]:

$$-9 \times 10^{-3} \leq \phi \leq 4 \times 10^{-3}, \quad (29)$$

with a 90 % C. L. Other limits on the mixing angle ϕ reported in the literature are given in the Refs. [17,18].

As was discussed in Ref. [10], $N \approx \sigma(\phi, \mu_{\nu_\tau}, d_{\nu_\tau})\mathcal{L}$. Using the Poisson statistic [15,30], we require that $N \approx \sigma(\phi, \mu_{\nu_\tau}, d_{\nu_\tau})\mathcal{L}$ be less than 14, with $\mathcal{L} = 137 \text{ pb}^{-1}$, according to the data reported by the L3 Collaboration Ref. [15] and references therein. Taking this into consideration, we put a bound for the tau neutrino magnetic moment as a function of the ϕ mixing parameter with $d_{\nu_\tau} = 0$. We show the value of this bound for values of the ϕ parameter in Tables 1 and 2.

| ϕ | $\mu_{\nu_\tau}(10^{-6}\mu_B)$ | $d_{\nu_\tau}(10^{-17}\text{ecm})$ |
|--------|--------------------------------|------------------------------------|
| -0.009 | 4.48 | 8.64 |
| -0.005 | 4.44 | 8.56 |
| 0 | 4.40 | 8.49 |
| 0.004 | 4.37 | 8.43 |

Table 1. Bounds on the μ_{ν_τ} magnetic moment and d_{ν_τ} electric dipole moment for different values of the mixing angle ϕ before the Z_1 resonance, *i.e.*, $s \approx M_{Z_1}^2$.

These results are comparable with the bounds obtained in the references [10,11]. However, the derived bounds in Table 1 could be improved by including data from the entire Z_1 resonance as is shown in Table 2.

| ϕ | $\mu_{\nu_\tau}(10^{-6}\mu_B)$ | $d_{\nu_\tau}(10^{-17}\text{ecm})$ |
|--------|--------------------------------|------------------------------------|
| -0.009 | 3.37 | 6.50 |
| -0.005 | 3.34 | 6.44 |
| 0 | 3.31 | 6.38 |
| 0.004 | 3.28 | 6.32 |

Table 2. Bounds on the μ_{ν_τ} magnetic moment and d_{ν_τ} electric dipole moment for different values of the mixing angle ϕ in the Z_1 resonance, *i.e.*, $s = M_{Z_1}^2$.

The above analysis and comments can readily be translated to the electric dipole moment of the τ -neutrino with $\mu_{\nu_\tau} = 0$. The resulting bound for the electric dipole moment as a function of the ϕ mixing parameter is shown in Table 1.

The results in Table 2 for the electric dipole moment are in agreement with those found by the L3 Collaboration [15].

We end this section by plotting the total cross section in Fig. 2 as a function of the mixing angle ϕ for the bounds of the magnetic moment given in Tables 1, 2. We observe in Fig. 2 that for $\phi = 0$, we reproduce the data previously reported in the literature. Also, we observe that the total cross section increases constantly and reaches its maximum value for $\phi = 0.004$.

V. CONCLUSIONS

We have determined a bound on the magnetic moment and the electric dipole moment of a massive tau neutrino in the framework of a left-right symmetric model as a function of the mixing angle ϕ , as shown in Table 1 and Table 2.

Other upper limits on the tau neutrino magnetic moment reported in the literature are $\mu_{\nu_\tau} < 3.3 \times 10^{-6} \mu_B$ (90%*C.L.*) from a sample of e^+e^- annihilation events collected with the L3 detector at the Z_1 resonance corresponding to an integrated luminosity of 137 pb^{-1} [15]; $\mu_{\nu_\tau} \leq 2.7 \times 10^{-6} \mu_B$ (95%*C.L.*) at $q^2 = M_{Z_1}^2$ from measurements of the Z_1 invisible width at LEP [20]; $\mu_{\nu_\tau} < 1.83 \times 10^{-6} \mu_B$ (90%*C.L.*) from the analysis of $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ at the Z_1 -pole, in a class of E_6 inspired models with a light additional neutral vector boson [22]; from the order of $\mu_{\nu_\tau} < O(1.1 \times 10^{-6} \mu_B)$ Keiichi Akama *et al.* derive and apply model-independent bounds on the anomalous magnetic moments and the electric dipole moments of leptons and quarks due to new physics [21]. However, the limits from Ref. [21] are for the tau neutrino with an upper bound of $m_\tau < 18.2 \text{ MeV}$ which is a direct experimental limit at present.

It is pointed out in Ref. [21] however, that the upper limit on the mass of the electron neutrino and data from various neutrino oscillation experiments together imply that none of the active neutrino mass eigenstates is heavier than approximately 3 eV. In this case, limits from Ref. [21] improve by seven orders of magnitude. The limit $\mu_{\nu_\tau} < 5.4 \times 10^{-7} \mu_B$ (90% C.L.) is obtained at $q^2 = 0$ from a beam-dump experiment with assumptions on the D_s production cross section and its branching ratio into $\tau\nu_\tau$ [31], thus severely restricting the cosmological annihilation scenario [32]. Our results in Tables 1 and 2 confirm the bound obtained by the L3 Collaboration [15].

In the case of the electric dipole moment, other upper limits reported in the literature are [20,21]:

$$|d(\nu_\tau)| \leq 5.2 \times 10^{-17} e \text{ cm} \quad 95 \% \text{ C.L.}, \quad (30)$$

$$|d(\nu_\tau)| < O(2 \times 10^{-17} e \text{ cm}). \quad (31)$$

In summary, we conclude that the estimated bound for the tau neutrino magnetic moment and the electric dipole moment are almost independent of the experimental allowed values of the ϕ parameter of the model. In the limit $\phi = 0$, our bound takes the value previously reported in the Ref. [15]. In addition, the analytical and numerical results for the cross-section have never been reported in the literature before and could be of relevance for the scientific community.

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FIGURE CAPTIONS

Fig. 1 The Feynman diagrams contributing to the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$, in a left-right symmetric model.

Fig. 2 The total cross section for $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ as a function of ϕ and μ_{ν_τ} (Tables 1, 2).

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